PH16212, Homework 4

Deadline: Nov. 6, 2019

1. For the six-point Yang-Mills NHMV amplitude,

$$A(1^-2^-3^-4^+5^+6^+) = \frac{-i(\langle 1,5\rangle[4,5]+\langle 1,6\rangle[4,6])^3}{\langle 1,6\rangle\langle 5,6\rangle\ [2,3][3,4](\langle 1,5\rangle[1,2]+\langle 5,6\rangle[2,6])(s_{15}+s_{16}+s_{56})} + \frac{-i(\langle 1,3\rangle[1,6]+\langle 2,3\rangle[2,6])^3}{\langle 3,4\rangle\langle 4,5\rangle[1,2]\ [1,6](\langle 1,5\rangle\ [1,2]+\langle 5,6\rangle[2,6])(s_{12}+s_{16}+s_{26})}\,, \tag{1}$$

Explicitly calculate the soft limit for $p_3^\mu \to 0$.

2. Consider the one-loop massless pentagon integral with

$$D_1 = l^2, \quad D_2 = (l-p_1)^2, \quad D_3 = (l-p_1-p_2)^2, \quad D_4 = (l-p_1-p_2-p_3)^2 \quad D_4 = (l+p_5)^2 \tag{2}$$

with $p_i^2 = 0$, $i = 1, \ldots 5$. Use s_{12}, s_{23}, s_{34}, s_{45}, s_{15} as the *independent* kinematic variables.

- Find the two Symanzik polynomials U and F for this integral.
- Determine the Euclidean region for this integral.

3. Consider the two-loop massive sunset diagram with

$$D_1 = l_1^2 - m_1^2, \quad D_2 = l_2^2 - m_2^2, \quad D_3 = (l_2+l_1+p)^2 - m_3^2, \tag{3}$$

with $p^2 = s$.

- Find the two Symanzik polynomials U and F for this integral.
- Express the $(D-2)$-dimensional integral $G^{(D-2)}[1,1,1]$ as a linear combination of D-dimensional integrals for this diagram.